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Some Open Problems in the Theory of Computation as Questions about Two-Way Deterministic Pushdown Automaton Languages

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ABSTRACT

We considered some of the important unsolved problems in the theory of computation concerning the relationship between deterministic and nondeterministic computations, and between tape and time bounded computations. For each such problem we find an equivalent problem concerning two-way deterministic pushdown automaton languages.

0. Introduction. In the theory of computation we can find several open problems concerning the relationship between tape and time bounded computations and the relationship between deterministic and nondeterministic computations. One of them, the *LBA* problem ([18], [19], [10]), has been studied over ten years.

Each of these problems is of the following form: There are two classes of languages, $\mathscr{L}_A$ and $\mathscr{L}_B$, and two classes of machines, $\mathscr{M}_A$ and $\mathscr{M}_B$, which characterize them. In each case it is easy to prove that $\mathscr{L}_A \subseteq \mathscr{L}_B$ and the open problem is whether $\mathscr{L}_A$ equals $\mathscr{L}_B$. It is usually the case that there are languages in $\mathscr{L}_B$ that are easily shown to be there, but nobody can prove that they are not in $\mathscr{L}_A$. There is however still a chance that although $\mathscr{L}_B$ looks much richer, each machine in $\mathscr{M}_B$ can be simulated by some machine in $\mathscr{M}_A$, perhaps using clever tricks.

Unable to solve these open problems, researchers have recently reduced them to various other, seemingly simpler problems. One of the most common approaches is to construct a particular language L in $\mathscr{L}_B$ such that $L \in \mathscr{L}_A$ if and only if $\mathscr{L}_B = \mathscr{L}_A$. Thus, the question "can we simulate all machines in $\mathscr{M}_B$ by machines in $\mathscr{M}_A$?" is reduced to the question "can we accept L by a machine in $\mathscr{M}_A$?", which may appear easier. Moreover, the aim is to construct a candidate for language L as simple as possible and thus to reduce the original problem to the simplest form.

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